Lightweight Design in Mechanical Engineering

Problem 2. Lightweight Design of rods under shear loads

Matthias Kröger, Prof. Dr.-Ing. at TUBAF
Serhii Onyshchenko, PhD, assoc. prof. at DUT

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Problem 2

Two parts are fastened by **4 rivets (Fig. 1)**. Calculate lightweight design of the fastening by rivets as rods loaded with shear forces. Assume that the loading is distributed evenly among the rivets. Friction between fastened parts is absent.

Loading forces are **F = 2 kN**.

Thickness of fastened parts is **a = 10 mm**.

Minimum safety factor is $\mathbf{n_{min} = 3}$.

Permissible shear stress is $\boldsymbol{\tau_{perm}}$.

Consider hollow rivet design.

Consider materials with high strength-to-density ratio.

Perform cost comparison.

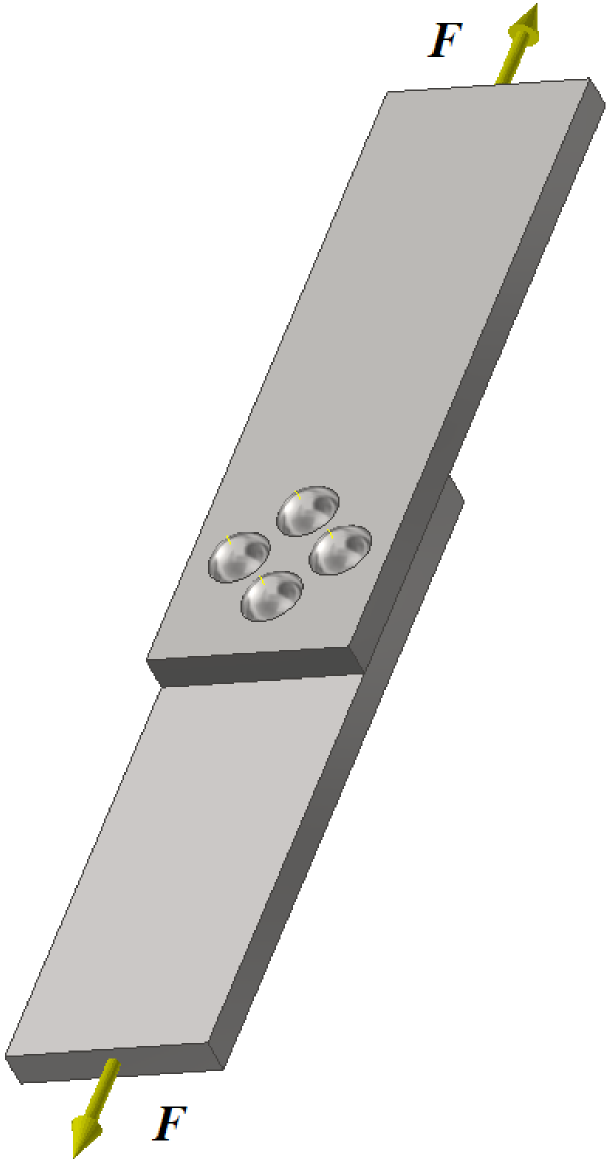

Fig. 1 – Loading scheme

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1. Consider solid rivets made of **stainless steel, copper, aluminum and titanium alloys**.

Stainless steel AISI 316; $\boldsymbol{\sigma_{yield.St} = 205}$ **MPa;**
Copper alloy – Brass (Cu–Zn); $\boldsymbol{\sigma_{yield.Br} = 144}$ **MPa;**
Aluminum 6061; $\boldsymbol{\sigma_{yield.Al} = 240}$ **MPa;**
Titanium Grade 5 (Ti-6Al-4V); $\boldsymbol{\sigma_{yield.Ti} = 790}$ **MPa.**

Assume $\boldsymbol{\tau_{perm} = 0.6\, \sigma_{yield} / n_{min}}$ for all materials.
$\boldsymbol{\tau_{perm.St} = 41}$ **MPa;**
$\boldsymbol{\tau_{perm.Br} = 28.8}$ **MPa;**
$\boldsymbol{\tau_{perm.Al} = 48}$ **MPa;**
$\boldsymbol{\tau_{perm.Ti} = 158}$ **MPa.**

2. Construct a free-body diagram (FBD) to determine **shear forces V** in rivets **(Fig. 2)**.

From equilibrium condition $\boldsymbol{\Sigma F_i = 0}$,

$$\mathbf{F - 4 \cdot V = 0; \quad V = F / 4 = 0.5\ kN.}$$

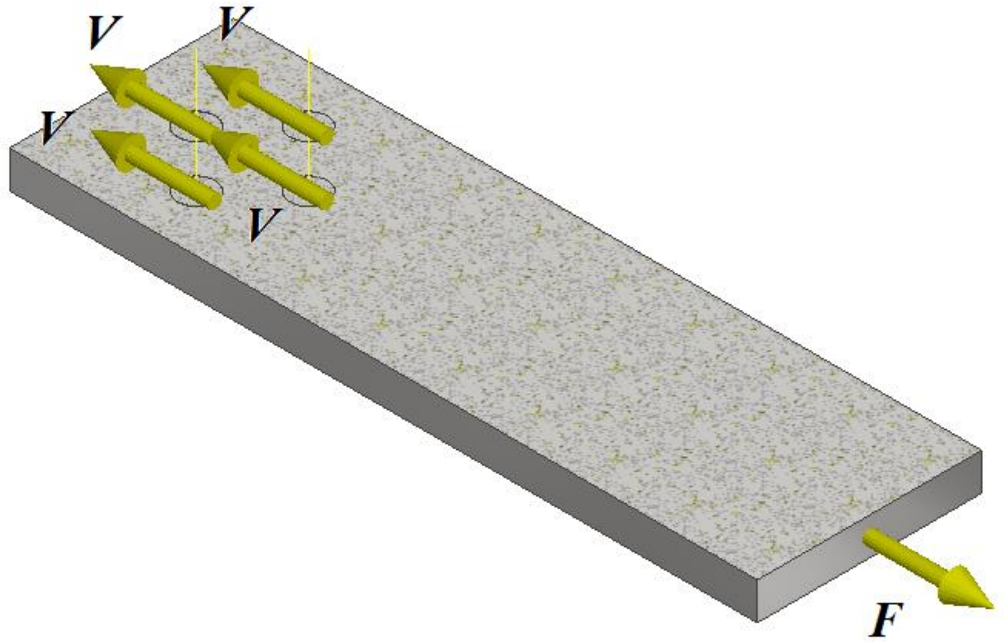

Fig. 2 – FBD for determining shear forces

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3. Determine minimum rivet **cross-section area A_{min} (Fig. 3),** which satisfies the safety margin **n_{min} = 3.**

$A = V / \tau_{perm}$;

$A_{min.St} = V / \tau_{perm.St} = 12.2\ mm^2$;

$A_{min.Br} = V / \tau_{perm.Br} = 17.4\ mm^2$;

$A_{min.Al} = V / \tau_{perm.Al} = 10.4\ mm^2$;

$A_{min.Ti} = V / \tau_{perm.Ti} = 3.2\ mm^2$;

4. Calculate minimum **rivet diameters d_{min}**

$d_{min.St} = (4 \cdot A_{min.St} / \pi)^{1/2} = 3.94\ mm$;

$d_{min.Br} = (4 \cdot A_{min.Br} / \pi)^{1/2} = 4.7\ mm$;

$d_{min.Al} = (4 \cdot A_{min.Al} / \pi)^{1/2} = 3.64\ mm$;

$d_{min.Ti} = (4 \cdot A_{min.Ti} / \pi)^{1/2} = 2.0\ mm$.

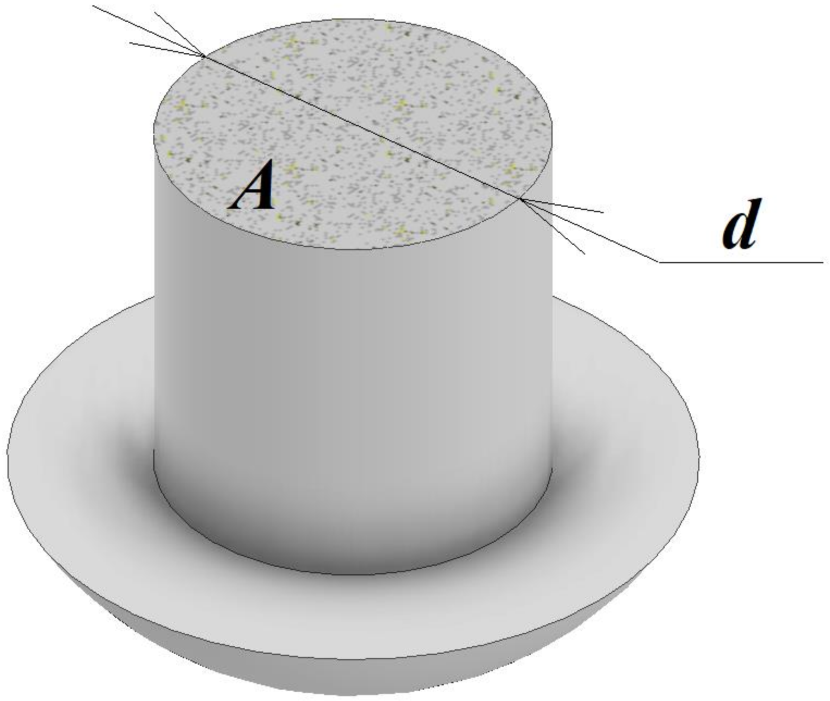

Fig. 3 – Solid rivet cross-section scheme

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5. Select **diameters of solid rivets (Fig. 4)**, which satisfy the safety margin **n_{min} = 3,** and calculate actual cross-section areas

$d_{act.St}$ = 4 mm; $A_{act.St} = [\pi \cdot (d_{act.St})^2] / 4 = 12.57\ mm^2$;
$d_{act.Br}$ = 5 mm; $A_{act.Br} = [\pi \cdot (d_{act.Br})^2] / 4 = 19.64\ mm^2$;
$d_{act.Al}$ = 4 mm; $A_{act.Al} = [\pi \cdot (d_{act.Al})^2] / 4 = 12.57\ mm^2$;
$d_{act.Ti}$ = 3 mm; $A_{act.Ti} = [\pi \cdot (d_{act.Ti})^2] / 4 = 7.07\ mm^2$;

6. Determine **actual tangent stresses τ_{act}** and **safety factors n_{act}** for selected diameters of rivets

$\tau_{act.St} = V / A_{act.St} = 39.79$ MPa; $n_{act.St} = \tau_{perm.St} \cdot n_{min} / \tau_{act.St} = 3.09$;
$\tau_{act.Br} = V / A_{act.Br} = 25.47$ MPa; $n_{act.Br} = \tau_{perm.Br} \cdot n_{min} / \tau_{act.Br} = 3.39$;
$\tau_{act.Al} = V / A_{act.Al} = 39.79$ MPa; $n_{act.Al} = \tau_{perm.Al} \cdot n_{min} / \tau_{act.Al} = 3.62$;
$\tau_{act.Ti} = V / A_{act.Ti} = 70.74$ MPa; $n_{act.Ti} = \tau_{perm.Ti} \cdot n_{min} / \tau_{act.Ti} = 6.7$.

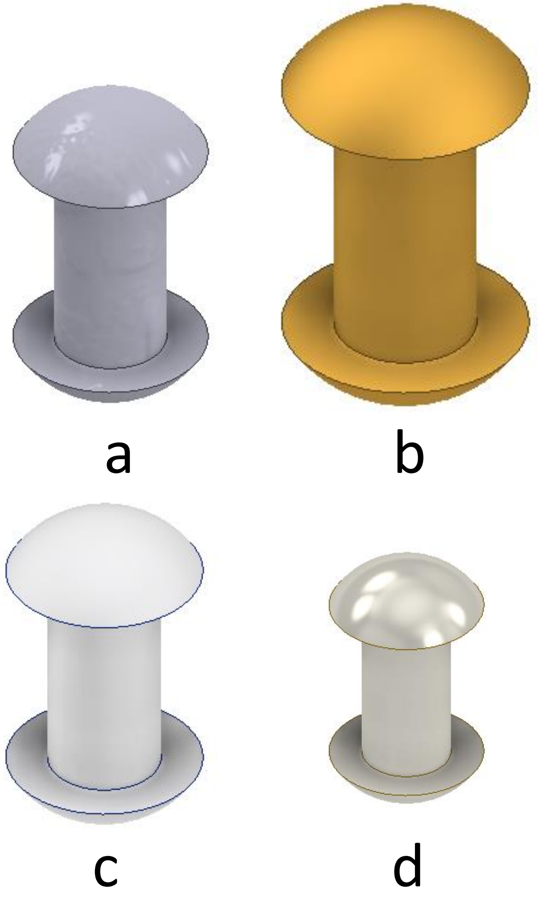

Fig. 4 – Solid rivets:
a – steel, b – brass,
c – aluminum, d – titanium

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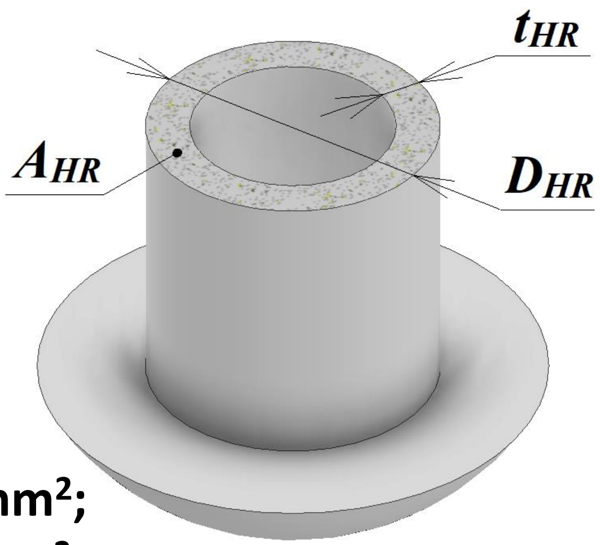

Fig. 5 – Hollow rivet (HR) cross-section

7. Calculate a **hollow (tubular) rivet design (Fig. 5)**. Usually, the **opening** in a hollow rivet **is 70%** of nominal **rivet diameter**. Select hollow rivet **outside diameters D_{HR}** and verify selection if the **cross-section area A_{HR}** is more than the minimum A_{min}.

$D_{HR.St}$ = 6 mm; $t_{HR.St} = 0.15 \cdot D_{HR.St}$ = 0.9 mm; $A_{HR.St} = \pi (D_{HR.St} \cdot t_{HR.St} - t_{HR.St}^2)$ = 14.42 mm²;

$D_{HR.Br}$ = 8 mm; $t_{HR.Br} = 0.15 \cdot D_{HR.Br}$ = 1.2 mm; $A_{HR.Br} = \pi (D_{HR.Br} \cdot t_{HR.Br} - t_{HR.Br}^2)$ = 18.1 mm²;

$D_{HR.Al}$ = 5 mm; $t_{HR.Al} = 0.15 \cdot D_{HR.Al}$ = 0.75 mm; $A_{HR.Al} = \pi (D_{HR.Al} \cdot t_{HR.Al} - t_{HR.Al}^2)$ = 12.37 mm²;

$D_{HR.Ti}$ = 2 mm; $t_{HR.Ti} = 0.15 \cdot D_{HR.Ti}$ = 0.3 mm; $A_{HR.Ti} = \pi (D_{HR.Ti} \cdot t_{HR.Ti} - t_{HR.Ti}^2)$ = 5.37 mm².

8. Determine **actual tangent stresses τ_{HR}** and **safety factors n_{HR}**

$\tau_{HR.St} = V / A_{HR.St}$ = 34.67 MPa; $n_{HR.St} = \tau_{perm.St} \cdot n_{min} / \tau_{HR.St}$ = 3.55;

$\tau_{HR.Br} = V / A_{HR.Br}$ = 27.63 MPa; $n_{HR.Br} = \tau_{perm.Br} \cdot n_{min} / \tau_{HR.Br}$ = 3.13;

$\tau_{HR.Al} = V / A_{HR.Al}$ = 40.42 MPa; $n_{HR.Al} = \tau_{perm.Al} \cdot n_{min} / \tau_{HR.Al}$ = 3.56;

$\tau_{HR.Ti} = V / A_{HR.Ti}$ = 93.07 MPa; $n_{HR.Ti} = \tau_{perm.Ti} \cdot n_{min} / \tau_{HR.Ti}$ = 5.1.

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9. Perform volume calculation for **actual solid and hollow rivets** made of **Stainless Steel**, **Brass**, **Aluminum** and **Titanium**. Average **rivet head volume** is **40%** and **100%** of pin volume for solid and hollow rivets, respectively.

$V_{St} = A_{act.St} \cdot a \cdot 1.4 = 175.93\ mm^3$; $V_{HR.St} = A_{HR.St} \cdot a \cdot 2 = 288.40\ mm^3$;

$V_{Br} = A_{act.Br} \cdot a \cdot 1.4 = 274.89\ mm^3$; $V_{HR.Br} = A_{HR.Br} \cdot a \cdot 2 = 361.91\ mm^3$;

$V_{Al} = A_{act.Al} \cdot a \cdot 1.4 = 175.93\ mm^3$; $V_{HR.Al} = A_{HR.Al} \cdot a \cdot 2 = 247.40\ mm^3$;

$V_{Ti} = A_{act.Ti} \cdot a \cdot 1.4 = 98.96\ mm^3$; $V_{HR.Ti} = A_{HR.Ti} \cdot a \cdot 2 = 107.44\ mm^3$.

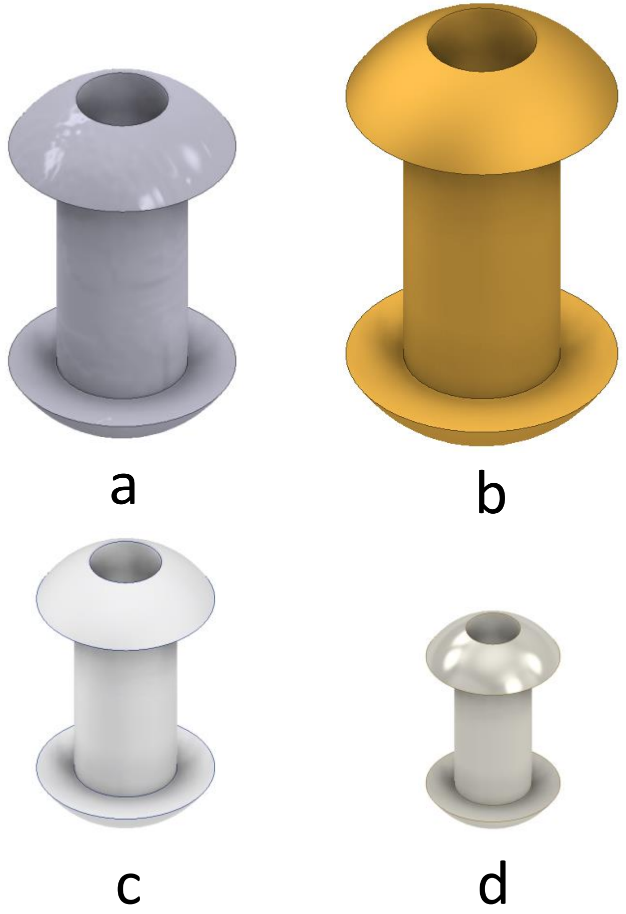

Fig. 6 – Hollow rivets (HR): a – steel, b – brass, c – aluminum, d – titanium

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10. Densities of materials are

Stainless steel AISI 316	ρ_{St} = 8000 kg/m³;	Brass	ρ_{Br} = 8550 kg/m³;
Aluminum 6061	ρ_{Al} = 2700 kg/m³;	Titanium Grade 5	ρ_{Ti} = 4430 kg/m³.

11. Calculate **mass and weight of 4 solid rivets** made of various materials

$m_{St} = 4 \cdot \rho_{St} \cdot V_{St} = 0.0056$ kg; **$G_{St} = m_{St} \cdot g = 0.055$ N;**
$m_{Br} = 4 \cdot \rho_{Br} \cdot V_{Br} = 0.0094$ kg; **$G_{Br} = m_{Br} \cdot g = 0.092$ N;**
$m_{Al} = 4 \cdot \rho_{Al} \cdot V_{Al} = 0.0019$ kg; **$G_{Al} = m_{Al} \cdot g = 0.019$ N;**
$m_{Ti} = 4 \cdot \rho_{Ti} \cdot V_{Ti} = 0.0018$ kg; **$G_{Ti} = m_{Ti} \cdot g = 0.002$ N;**

and hollow rivets

$m_{HR.St} = 4 \cdot \rho_{St} \cdot V_{HR.St} = 0.0092$ kg; **$G_{HR.St} = m_{HR.St} \cdot g = 0.090$ N;**
$m_{HR.Br} = 4 \cdot \rho_{Br} \cdot V_{HR.Br} = 0.0124$ kg; **$G_{HR.Br} = m_{HR.Br} \cdot g = 0.121$ N;**
$m_{HR.Al} = 4 \cdot \rho_{Al} \cdot V_{HR.Al} = 0.0027$ kg; **$G_{HR.Al} = m_{HR.Al} \cdot g = 0.026$ N;**
$m_{HR.Ti} = 4 \cdot \rho_{Ti} \cdot V_{HR.Ti} = 0.0019$ kg; **$G_{HR.Ti} = m_{HR.Ti} \cdot g = 0.019$ N.**

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12. Perform cost calculations. Prices for materials per kg are

Stainless steel	p_{St} = 2.67 €/kg;
Brass	p_{Br} = 8.4 €/kg;
Aluminum 6061	p_{Al} = 5.97 €/kg;
Titanium Grade 5	p_{Ti} = 2.34 €/kg.

13. Calculate material costs

$MC_{St} = m_{St} \cdot p_{St} = 0.015$ €;

$MC_{Br} = m_{Br} \cdot p_{Br} = 0.079$ €;

$MC_{Al} = m_{Al} \cdot p_{Al} = 0.004$ €;

$MC_{Ti} = m_{Ti} \cdot p_{Ti} = 0.01$ €;

$MC_{HR.St} = m_{HR.St} \cdot p_{St} = 0.025$ €;

$MC_{HR.Br} = m_{HR.Br} \cdot p_{Br} = 0.104$ €;

$MC_{HR.Al} = m_{HR.Al} \cdot p_{Al} = 0.006$ €;

$MC_{HR.Ti} = m_{HR.Ti} \cdot p_{Ti} = 0.011$ €.

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Conclusion.

Solid rivets under shear loads **are more lightweight** than equally safe **hollow rivets** made of the same material.

Therefore, when calculating material costs, solid rivets are less expensive. For stainless steel – by 67%, for brass – by 32%, for aluminum – by 43%, for titanium – by 10%. This is mostly due to the fact that **rivet head volume is larger** for hollow design. The safety margins are sufficient for all cases.

Aluminum rivets have the lowest cost out of all materials.

Thank you for your attention!

Matthias Kröger, Prof. Dr.-Ing. at TUBAF

Serhii Onyshchenko, PhD, assoc. prof. at DUT