Lightweight Design in Mechanical Engineering

Problem 3. Lightweight Design of planar trusses

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Problem 3

Calculate lightweight design of a planar truss **(Fig. 1)** loaded with external forces.

Ensure minimum safety margin $\mathbf{n_{min} = 6}$ for all members.

Truss material is structural steel **S235J2**.

Density is $\boldsymbol{\rho_{St}}$ **= 7850 kg/m³**.

Loading forces are $\mathbf{F_1 = 10\ kN}$, $\mathbf{F_2 = 20\ kN}$, $\mathbf{F_3 = 30\ kN}$.

Truss dimensions are **a = 1 m, h = 3 m.**

Permissible stress in members is $\boldsymbol{\sigma_{perm}}$.

Consider a material with high strength-to-density ratio.

Perform material cost comparison.

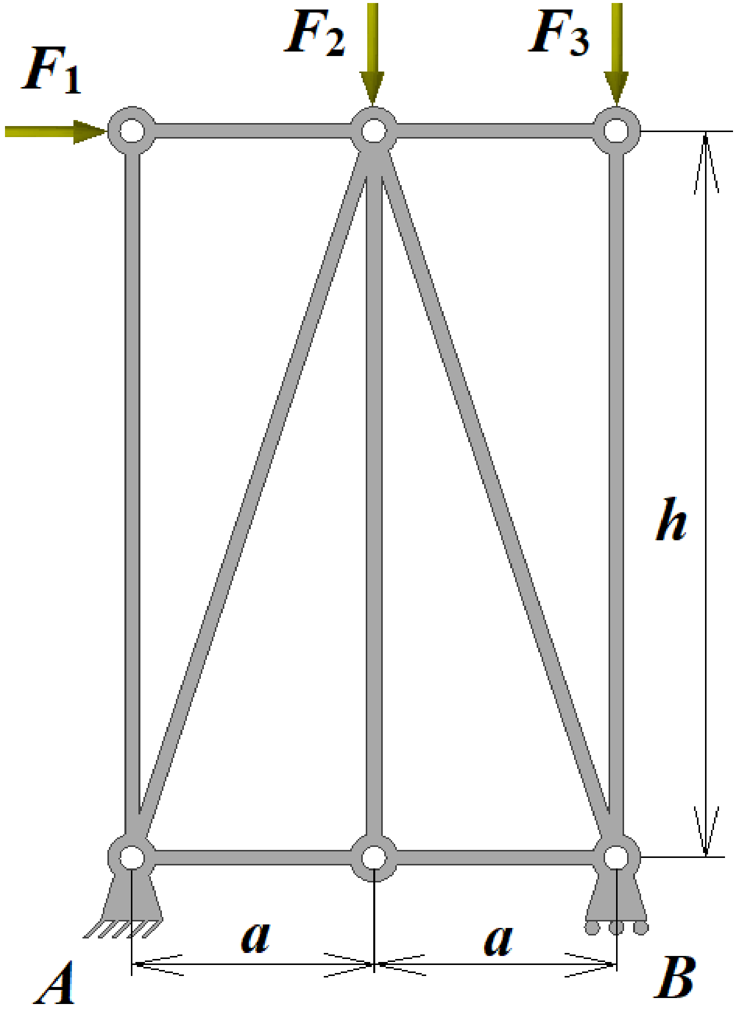

Fig. 1 – Truss scheme

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1. Construct equations of equilibrium **(Fig. 2)**

$\mathbf{\Sigma F_{ix} = 0}$: $-R_{Ax} + F_1 = 0$;

$\mathbf{\Sigma F_{iy} = 0}$: $-R_{Ay} + R_B - F_2 - F_3 = 0$;

$\mathbf{\Sigma M_A(F_i) = 0}$: $R_B \cdot 2a - F_1 \cdot h - F_2 \cdot a - F_3 \cdot 2a = 0$.

2. Calculate reactions of constraints

$R_B = (F_1 \cdot h + F_2 \cdot a + F_3 \cdot 2a)/2a =$ **55 kN**;

$R_{Ay} = R_B - F_2 - F_3 =$ **5 kN**;

$R_{Ax} = F_1 =$ **10 kN**.

Check the results

$\mathbf{\Sigma M_E(F_i) = 0}$: $R_{Ay} \cdot a - R_{Ax} \cdot h - F_3 \cdot a + R_B \cdot a = 0$

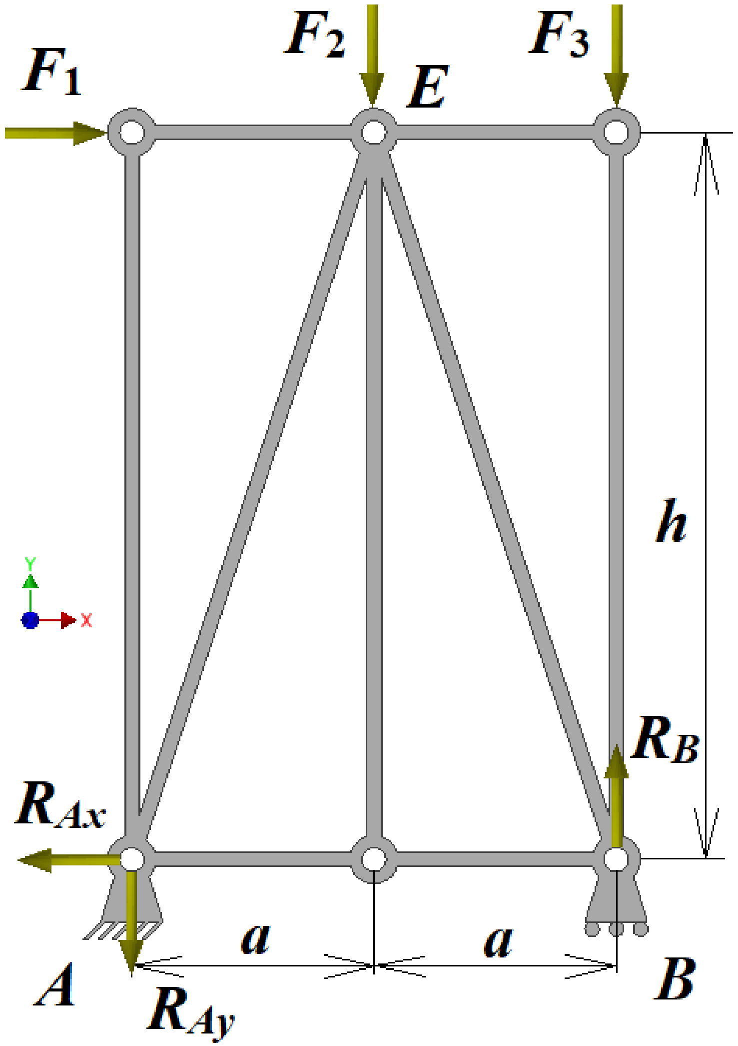

Fig. 2 – Calculation scheme

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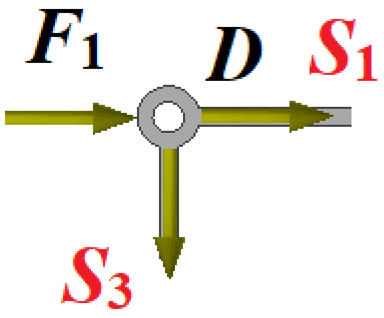
3. Use method of joints **(Fig. 3)** to determine forces in members

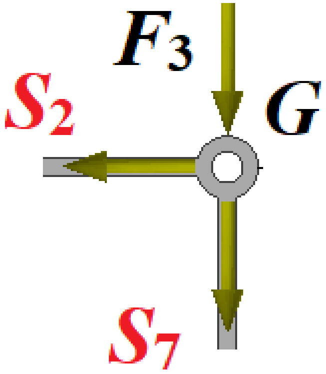
from **joint D**

$\mathbf{S_1 = -F_1 = -10\ kN;\ S_3 = 0;}$



from **joint G**

$\mathbf{S_2 = 0;\ S_7 = -F_3 = -30\ kN;}$



joint B

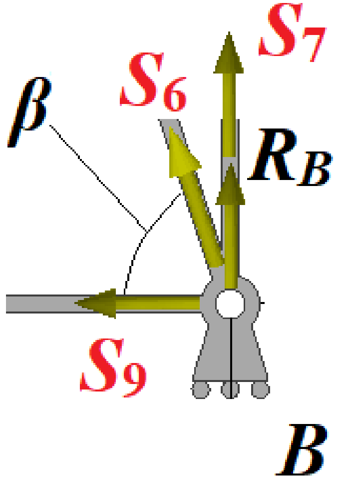
$\beta = \text{atan}(h/a) = 71.6$ deg;

$\mathbf{\Sigma F_{ix} = 0}: -S_9 - S_6 \cos(\beta) = 0;$

$\mathbf{\Sigma F_{iy} = 0}: S_6 \cdot \sin(\beta) + S_7 + R_B = 0;$

$\mathbf{S_6 = -26.4\ kN;}$

$\mathbf{S_9 = 8.3\ kN.}$



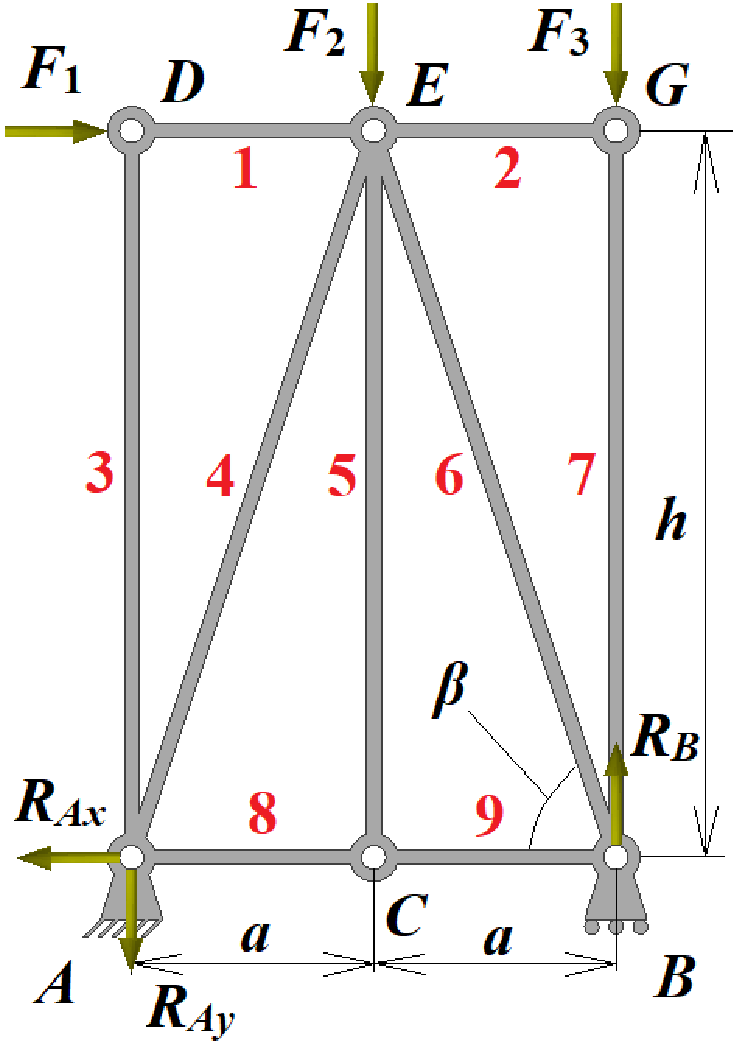


Fig. 3 – Joints and members

Problem 3

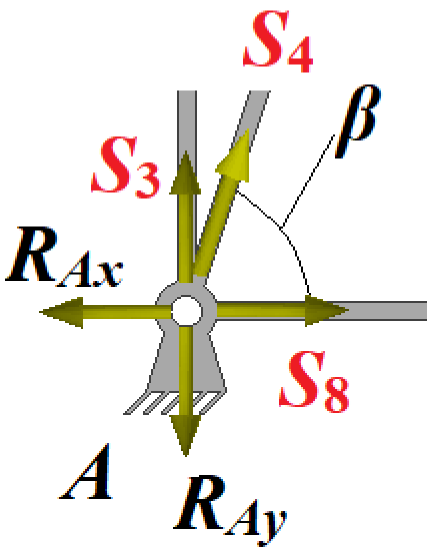
joint A

ΣF_{ix} = 0: $-R_{Ax} + S_8 + S_4\cdot\cos(\beta) = 0$;

ΣF_{iy} = 0: $S_3 + S_4\cdot\sin(\beta) - R_{Ay} = 0$;

S_8 = 8.33 kN;

S_4 = 5.27 kN.

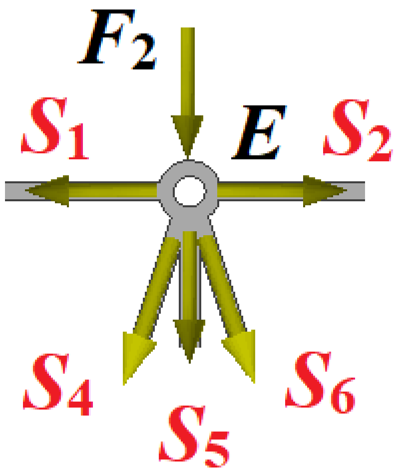


joint E

ΣF_{iy} = 0:

$-S_4\cdot\sin(\beta) - S_5 - S_6\cdot\sin(\beta) - F_2 = 0$;

S_5 = 0.



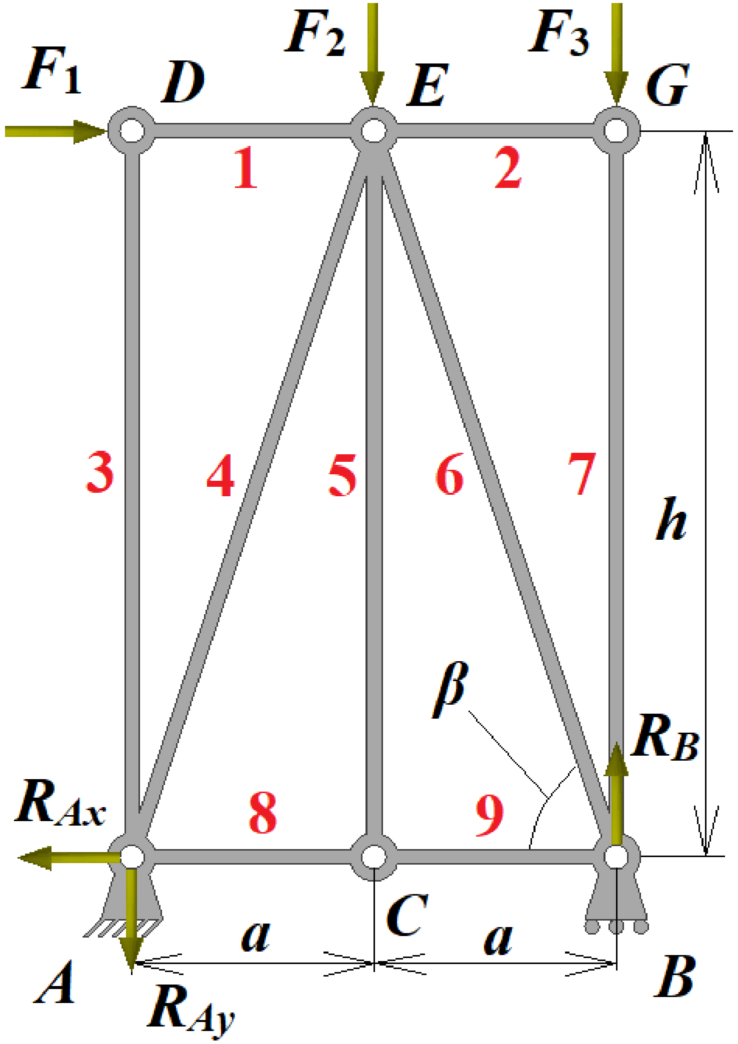


Fig. 3 – Joints and members

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4. Use method of sections to check the results (Figs. 4, 5)

Section through members 1, 4, 8

$\Sigma M_E(F_i) = 0$:

$R_{Ay} \cdot a - R_{Ax} \cdot h + S_8 \cdot h = 0;$

$\Sigma M_A(F_i) = 0$:

$-F_1 \cdot h - S_1 \cdot h = 0;$

$\Sigma F_{iy} = 0$:

$-R_{Ay} + S_4 \cdot \sin(\beta) = 0.$

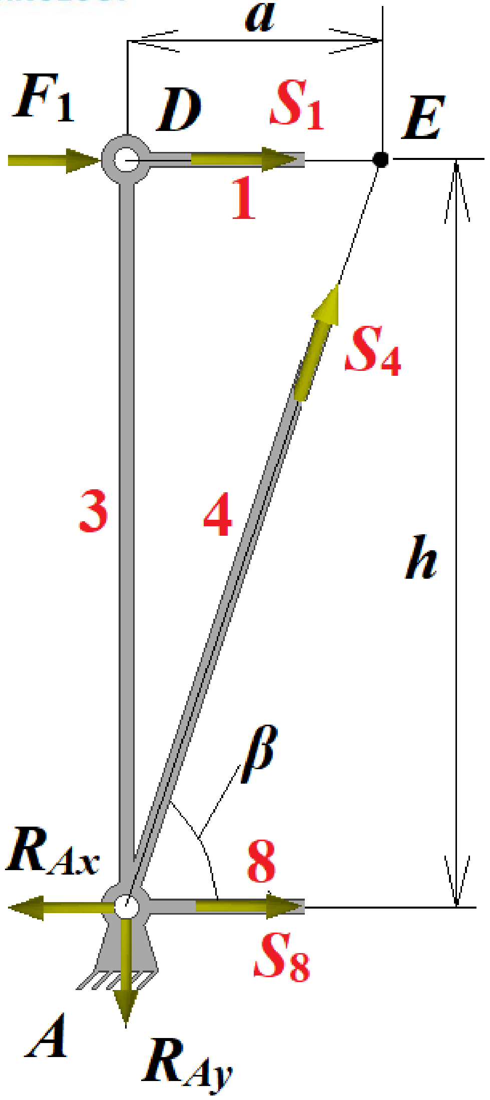

Fig. 4 – Section #1

Section through members 2, 6, 9

$\Sigma M_E(F_i) = 0$:

$-F_3 \cdot a + R_B \cdot a - S_9 \cdot h = 0;$

$\Sigma M_B(F_i) = 0$:

$S_2 \cdot h = 0;$

$\Sigma F_{iy} = 0$:

$-F_3 + R_B + S_6 \cdot \sin(\beta) = 0.$

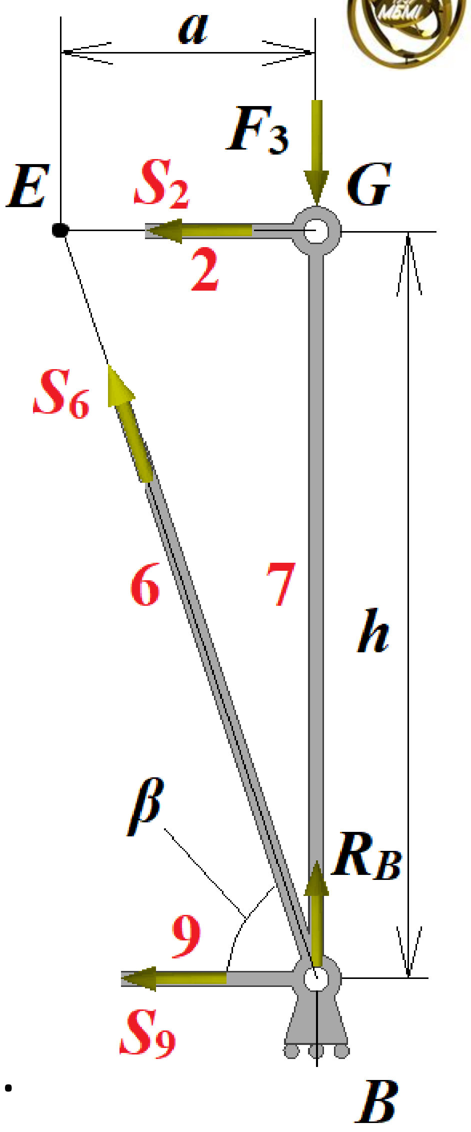

Fig. 5 – Section #2

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5. Determine minimum **cross-section areas** and **diameters** for **round steel members**

Yield strength is $\sigma_{yield.St}$ **= 235 MPa**;

Permissible stress $\sigma_{perm.St} = \sigma_{yield.St} / n_{min}$ **= 39.17 MPa**.

$A_{1.min} = |S_1| / \sigma_{perm.St} = 255.4\ mm^2$; $d_{1.min} = (4 \cdot A_{1.min} / \pi)^{1/2} = 18\ mm$;

$A_{2.min} = |S_2| / \sigma_{perm.St} = 0\ mm^2$; $d_{2.min} = (4 \cdot A_{2.min} / \pi)^{1/2} = 0\ mm$;

$A_{3.min} = |S_3| / \sigma_{perm.St} = 0\ mm^2$; $d_{3.min} = (4 \cdot A_{3.min} / \pi)^{1/2} = 0\ mm$;

$A_{4.min} = |S_4| / \sigma_{perm.St} = 134.6\ mm^2$; $d_{4.min} = (4 \cdot A_{4.min} / \pi)^{1/2} = 13.1\ mm$;

$A_{5.min} = |S_5| / \sigma_{perm.St} = 0\ mm^2$; $d_{5.min} = (4 \cdot A_{5.min} / \pi)^{1/2} = 0\ mm$;

$A_{6.min} = |S_6| / \sigma_{perm.St} = 672.8\ mm^2$; $d_{6.min} = (4 \cdot A_{6.min} / \pi)^{1/2} = 29.3\ mm$;

$A_{7.min} = |S_7| / \sigma_{perm.St} = 766\ mm^2$; $d_{7.min} = (4 \cdot A_{7.min} / \pi)^{1/2} = 31.2\ mm$;

$A_{8.min} = |S_8| / \sigma_{perm.St} = 212.8\ mm^2$; $d_{8.min} = (4 \cdot A_{8.min} / \pi)^{1/2} = 16.5\ mm$;

$A_{9.min} = |S_9| / \sigma_{perm.St} = 212.8\ mm^2$; $d_{9.min} = (4 \cdot A_{9.min} / \pi)^{1/2} = 16.5\ mm$.

Set zero values as the smallest value **(member 4)**

$A_{2.min} = A_{3.min} = A_{5.min} =$
$= A_{4.min} = 134.6\ mm^2$;

$d_{2.min} = d_{3.min} = d_{5.min} =$
$= d_{4.min} = 13.1\ mm$.

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6. Calculate length of **round steel members**

$L_1 = a = 1$ m;

$L_2 = a = 1$ m;

$L_3 = h = 3$ m;

$L_4 = a/\cos(\beta) = 3.16$ m;

$L_5 = h = 3$ m;

$L_6 = a/\cos(\beta) = 3.16$ m;

$L_7 = h = 3$ m;

$L_8 = a = 1$ m;

$L_9 = a = 1$ m;

7. Determine total length of all members

$L_{tot} = L_1 + L_2 + \ldots + L_9 = 19.33$ m.

8. Find minimum volume

$V_{min} = L_1 \cdot A_{1.min} + L_2 \cdot A_{2.min} + \ldots + L_9 \cdot A_{9.min} = 0.0065$ m^3.

9. Calculate minimum mass if $\rho_{St} = 7850$ kg/m^3

$m_{min.St} = V_{min} \cdot \rho_{St} = 50.82$ kg.

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10. Consider a case when **all members are of the same cross-section area** and $\mathbf{n_{min}}$ **= 6,** which means the area for all members is determined by the area of the most loaded member – **number 7**.

Calculate the volume when all members have the area $\mathbf{A_{7.min}}$

$\mathbf{V_{min.A7} = A_{7.min} \cdot L_{tot} = 0.0148\ m^3}$**.**

Determine mass for this case

$\mathbf{m_{min.St.A7} = V_{min.A7} \cdot \rho_{St} = 116.19\ kg}$**.**

This value is **2.29 times more** than in **step 9**.

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11. Perform calculations for **Aluminum 6061**, ρ_{Al} **= 2700 kg/m³.**
Yield strength is $\sigma_{yield.St}$ **= 240 MPa**;
Permissible stress $\sigma_{perm.St} = \sigma_{yield.St} / n_{min}$ **= 40 MPa**.

12. **Steel S235J2** has **strength/density ratio** of $\sigma_{yield.St} / \rho_{St}$ **=** $29936\ m^2s^{-2}$ and **aluminum 6061** has $\sigma_{yield.Al} / \rho_{Al}$ **=** $88888\ m^2s^{-2}$. Therefore, **2.97 times more**.

13. Repeat steps 5, 8, 9, 10 for **round aluminum members**

$V_{min.Al} = 0.0063\ m^3$ (for steel it is $0.0065\ m^3$);

$m_{min.Al} = V_{min.Al} \cdot \rho_{Al} = 17.12$ **kg** (for steel it is **50.82 kg** or **2.97 times more);**

$V_{min.Al.A7} = A_{7.min.Al} \cdot L_{tot} = 0.0145\ m^3$ (for steel it is $0.0148\ m^3$);

$m_{min.Al.A7} = V_{min.Al.A7} \cdot \rho_{Al} = 39.13$ **kg.**

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14. Perform material cost calculations.

Price of **Steel S235J2** is **0.728 €/kg;**

price of **Aluminum 6061** is **5.97 €/kg.**

$$MC_{min.St} = p_{St} \cdot m_{min.St} = 37.0\ €;$$

$$MC_{min.Al} = p_{Al} \cdot m_{min.Al} = 102.2\ €;$$

$$MC_{min.St.A7} = p_{St} \cdot m_{min.St.A7} = 84.6\ €;$$

$$MC_{min.Al.A7} = p_{Al} \cdot m_{min.Al.A7} = 233.6\ €.$$

Conclusion.

Aluminum planar truss is almost **3 times lighter** than a steel truss under external loads and equal safety margins. But it is **2.8 times more expensive**.

Thank you for your attention!

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