Lightweight Design in Mechanical Engineering

Problem 1. Lightweight Design of rods under tensile and compressive loads

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Problem 1

Calculate lightweight design of a rod loaded with static axial tensile and compressive forces **(Fig. 1)**.

Minimum safety factor is $\mathbf{n_{min} = 3}$.

Permissible stress is $\boldsymbol{\sigma_{perm}}$.

Forces are $\mathbf{F_1 = 10\ kN}$ and $\mathbf{F_2 = 28\ kN}$.

Dimensions are **a = 1 m, b = 2 m**.

Consider hollow rod design.

Consider materials with high strength-to-density ratio.

Perform material cost comparison.

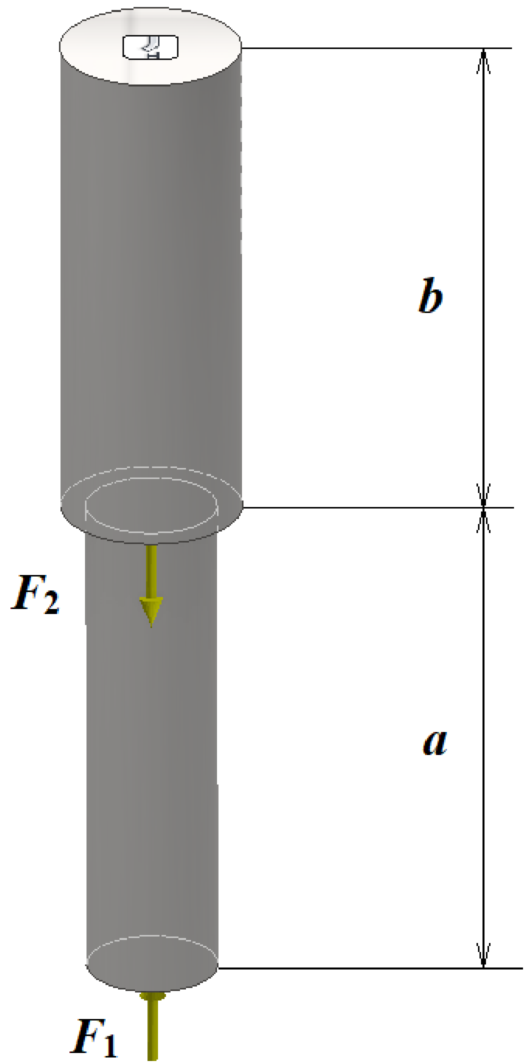

Fig. 1 – Loading scheme

Problem 1

1. Consider a **circular-shaped solid rod (round bar)** made of structural steel **S235J2 (1.0117)** according to the standard EN 10027-2.

Yield strength is $\boldsymbol{\sigma_{yield}}$ **= 235 MPa = 235 000 kPa;**
Permissible stress $\boldsymbol{\sigma_{perm}}$ **=** $\boldsymbol{\sigma_{yield}}$ **/** $\boldsymbol{n_{min}}$ **= 78.333 MPa = 78 333 kPa;**
Density is $\boldsymbol{\rho_{St}}$ **= 7850 kg/m³.**

2. Create free-body diagrams (FBD) **(Figs. 2 and 3)** and calculate internal tensile forces.

$$N_1 = -F_1 = -10 \text{ kN};$$

$$N_2 = -F_1 + F_2 = 18 \text{ kN}.$$

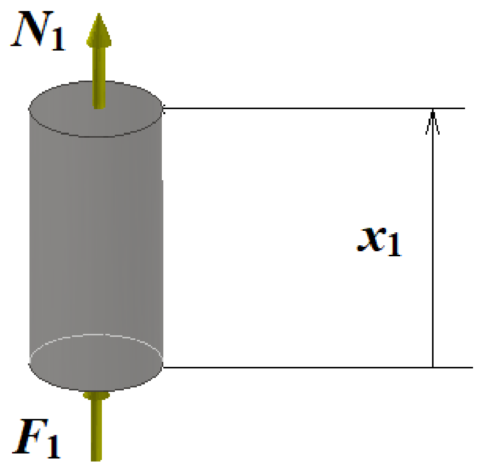

Fig. 2 – FBD #1

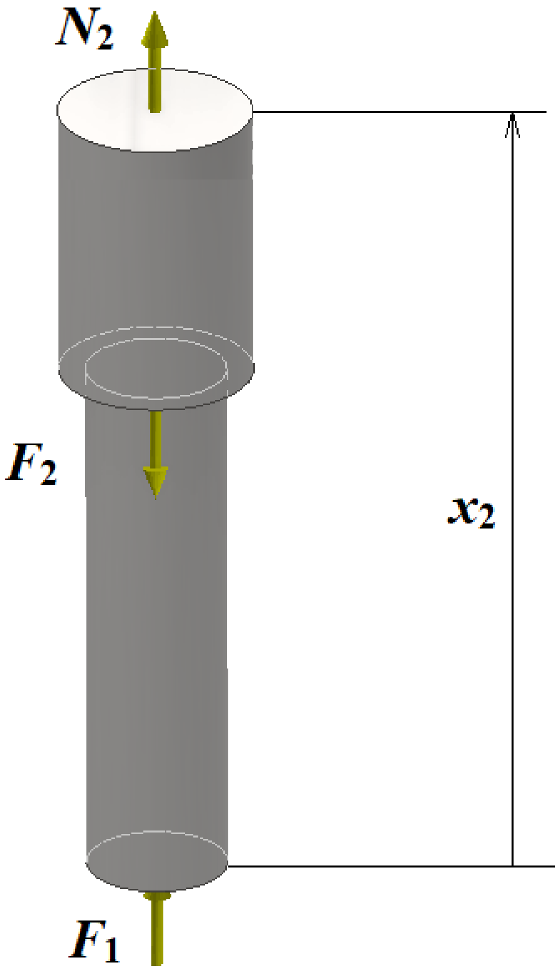

Fig. 3 – FBD #2

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3. Determine minimum cross-sectional area, which satisfies the safety factor $\mathbf{n_{min} = 3}$.

$$A_{1.min} = |N_1| / \sigma_{perm} = |-10\ kN| / 78\ 333\ kPa = 1.277 \cdot 10^{-4}\ m^2;$$
$$A_{2.min} = N_2 / \sigma_{perm} = 18\ kN / 78\ 333\ kPa = 2.298 \cdot 10^{-4}\ m^2.$$

4. Calculate minimum diameters **(Figs. 4 and 5)** of **circular-shaped solid rods (round bars)**.

$$d_{1.min} = (4 \cdot A_{1.min} / \pi)^{1/2} = 0.0128\ m = 12.8\ mm;$$
$$d_{2.min} = (4 \cdot A_{2.min} / \pi)^{1/2} = 0.0171\ m = 17.1\ mm.$$

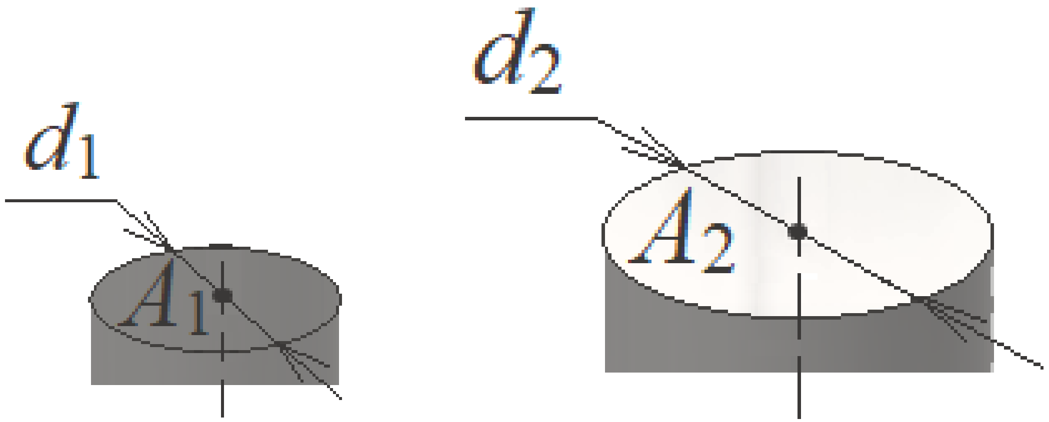

Fig. 4 – Diameter 1 Fig. 5 – Diameter 2

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5. Pick **round bars** produced according to the European Standard DIN EN 10060 : 2003.
$d_{1.act}$ = 13 mm; $d_{2.act}$ = 18 mm.

6. Determine actual **round bar cross-sectional areas**
$A_{1.act} = [\pi \cdot (d_{1.act})^2] / 4 = 1.327 \cdot 10^{-4}$ m²; $A_{2.act} = [\pi \cdot (d_{2.act})^2] / 4 = 2.545 \cdot 10^{-4}$ m²;

7. Calculate actual safety margins of **round bars**

$\sigma_{1.act} = |N_1| / A_{1.act}$ = 75.34 MPa; $\sigma_{2.act} = N_2 / A_{2.act}$ = 70.74 MPa;
$n_{1.act} = \sigma_{yield} / \sigma_{1.act}$ = 3.12; $n_{2.act} = \sigma_{yield} / \sigma_{2.act}$ = 3.32.

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8. Calculate an equally safe (**n_{min} = 3**) but hollow cross-sectional area **(round pipe) (Fig. 6)** considering **$A_{1.min}$** and **$A_{2.min}$** from before as minimum permissible values. **D** is outside pipe diameter, **t** is pipe wall thickness. According to ISO 3183 assume

D_1 = 26.7 mm, t_1 = 1.65 mm; D_2 = 48.3 mm, t_2 = 1.65 mm.

then pipe cross-sectional areas are

$$A_{1.RP} = \pi (D_1 \cdot t_1 - t_1^2) = 1.298 \cdot 10^{-4}\ m^2;$$
$$A_{2.RP} = \pi (D_2 \cdot t_2 - t_2^2) = 2.418 \cdot 10^{-4}\ m^2.$$

9. Calculate actual safety margins of **round pipes**

$$\sigma_{1.RP} = |N_1| / A_{1.RP} = 77.01\ MPa;\ \sigma_{2.RP} = N_2 / A_{2.RP} = 74.44\ MPa;$$
$$n_{1.RP} = \sigma_{yield} / \sigma_{1.RP} = 3.05;\ n_{2.act} = \sigma_{yield} / \sigma_{2.RP} = 3.16.$$

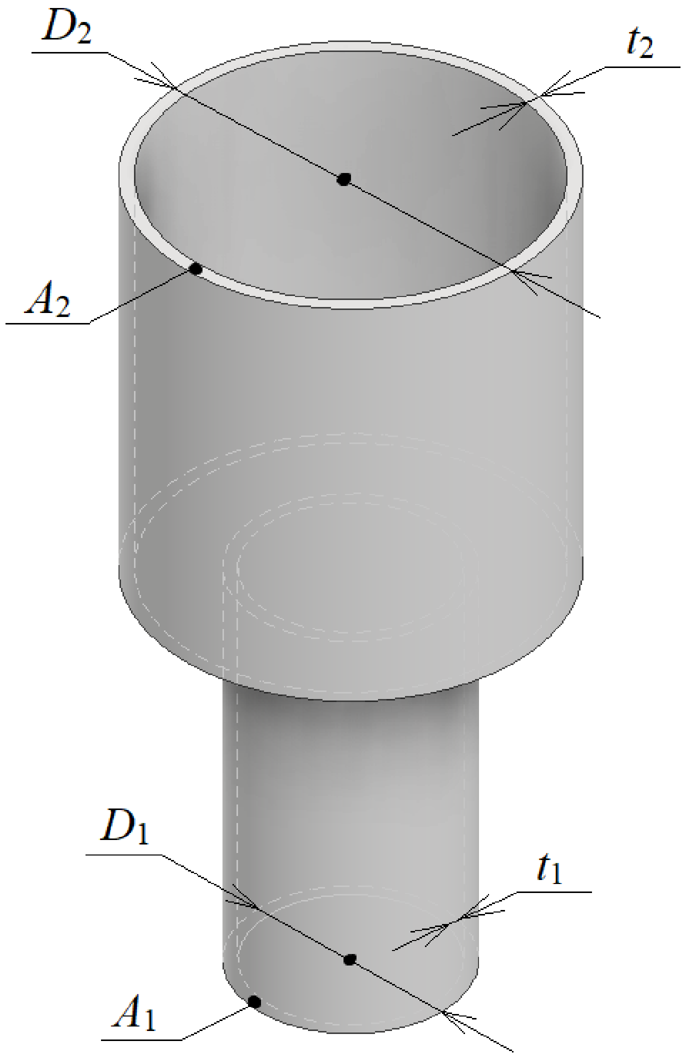

Fig. 6 – Round pipe scheme

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10. Lightweight design of loaded objects can be achieved by using materials with **high strength-to-density ratio**, like **Aluminum (Fig. 7) or Titanium alloys (Fig. 8)**.

Consider the alloy **Titanium Grade 5 (Ti-6Al-4V) and Aluminum 7075-T6** for the same loading conditions and the safety margin (**n_{min} = 3)**.

Yield strengths are $\sigma_{yield.Ti}$ **= 790 MPa** and $\sigma_{yield.Al}$ **= 500 MPa.**

Densities are ρ_{Ti} **= 4430 kg/m³** and ρ_{Al} **= 2810 kg/m³.**

Permissible stress for **Ti** $\sigma_{perm.Ti} = \sigma_{yield.Ti} / n_{min}$ **= 263.333 MPa;**

for **Al** $\sigma_{perm.Al} = \sigma_{yield.Al} / n_{min}$ **= 166.667 MPa.**

Fig. 7 – Aluminum alloy

Fig. 8 – Titanium alloy

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11. Repeat steps 3-7 for **Ti** and **Al**.

Minimum areas	$A_{1.min.Ti} = \lvert N_1 \rvert / \sigma_{perm.Ti} = 3.797 \cdot 10^{-5}$ m²; $A_{2.min.Ti} = N_2 / \sigma_{perm.Ti} = 6.835 \cdot 10^{-5}$ m²; $A_{1.min.Al} = \lvert N_1 \rvert / \sigma_{perm.Al} = 6 \cdot 10^{-5}$ m²; $A_{2.min.Al} = N_2 / \sigma_{perm.Al} = 1.08 \cdot 10^{-4}$ m²;
Minimum diameters	$d_{1.min.Ti}$ = 0.00695 m = 6.95 mm; $d_{2.min.Ti}$ = 0.00933 m = 9.33 mm; $d_{1.min.Al}$ = 0.00874 m = 8.74 mm; $d_{2.min.Al}$ = 0.01173 m = 11.73 mm;
Actual diameters	$d_{1.act.Ti}$ = 7 mm; $d_{2.act.Ti}$ = 10 mm; $d_{1.act.Al}$ = 9 mm; $d_{2.act.Al}$ = 12 mm;
Actual areas	$A_{1.act.Ti} = 3.848 \cdot 10^{-5}$ m²; $A_{2.act.Ti} = 7.854 \cdot 10^{-5}$ m²; $A_{1.act.Al} = 6.362 \cdot 10^{-5}$ m²; $A_{2.act.Al} = 1.131 \cdot 10^{-4}$ m²;
Actual stresses	$\sigma_{1.act.Ti}$ = 259.85 MPa; $\sigma_{2.act.Ti}$ = 229.18 MPa; $\sigma_{1.act.Al}$ = 157.19 MPa; $\sigma_{2.act.Al}$ = 159.16 MPa;
Actual safety margins	$n_{1.act.Ti}$ = 3.04; $n_{2.act.Ti}$ = 3.45; $n_{1.act.Al}$ = 3.18; $n_{2.act.Al}$ = 3.14.

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12. Perform volume calculation
for actual structures made of **Steel (Fig. 9), Ti (Fig. 10)** and **Al (Fig. 11)**.

$V_{St} = A_{1.act} \cdot a + A_{2.act} \cdot b = 6.417 \cdot 10^{-4}\ m^3$;
$V_{St.RP} = A_{1.RP} \cdot a + A_{2.RP} \cdot b = 6.135 \cdot 10^{-4}\ m^3$;
$V_{Ti} = A_{1.act.Ti} \cdot a + A_{2.act.Ti} \cdot b = 1.956 \cdot 10^{-4}\ m^3$;
$V_{Al} = A_{1.act.Al} \cdot a + A_{2.act.Al} \cdot b = 2.898 \cdot 10^{-4}\ m^3$.

13. Determine mass and weight of actual structures

$m_{St} = \rho_{St} \cdot V_{St} = 5.037$ kg; $G_{St} = m_{St} \cdot g = 49.364$ N;
$m_{St.RP} = \rho_{St} \cdot V_{St.RP} = 4.816$ kg; $G_{St.RP} = m_{St} \cdot g = 47.195$ N;
$m_{Ti} = \rho_{Ti} \cdot V_{Ti} = 0.866$ kg; $G_{Ti} = m_{Ti} \cdot g = 8.49$ N;
$m_{Al} = \rho_{Al} \cdot V_{Al} = 0.814$ kg. $G_{Al} = m_{Al} \cdot g = 7.981$ N.

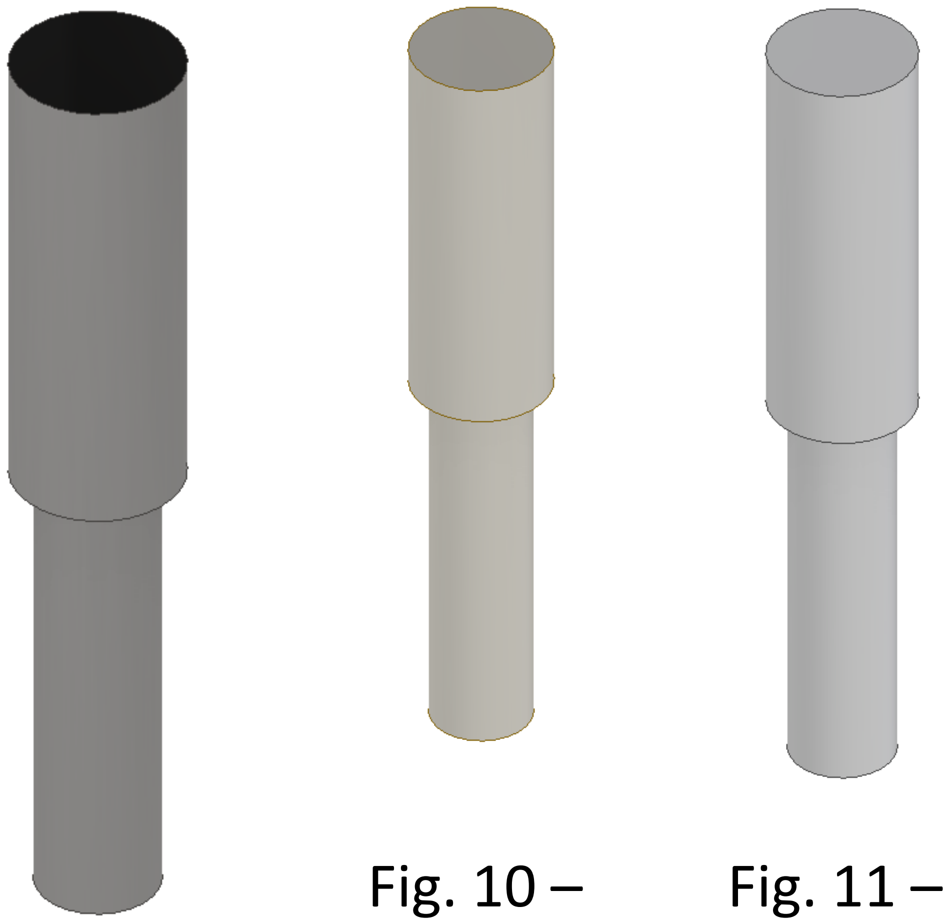

Fig. 9 – Steel model

Fig. 10 – Titanium model

Fig. 11 – Aluminum model

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14. Prices per kg of materials are

Steel round bar	p_{St} = 0.728 €/kg;
Steel round pipe	$p_{St.RP}$ = 0.759 €/kg;
Titanium round bar	p_{Ti} = 5.97 €/kg;
Aluminum round bar	p_{Al} = 2.955 €/kg.

15. Determine structure material cost (SMC)

$SMC_{St} = m_{St} \cdot p_{St} = 3.67$ €;
$SMC_{St.RP} = m_{St.RP} \cdot p_{St.RP} = 3.66$ €;
$SMC_{Ti} = m_{Ti} \cdot p_{Ti} = 5.17$ €;
$SMC_{Al} = m_{Al} \cdot p_{Al} = 2.41$ €.

Structure mass (for comparison)

$m_{St} = \rho_{St} \cdot V_{St} = 5.037$ kg;
$m_{St.RP} = \rho_{St} \cdot V_{St.RP} = 4.816$ kg;
$m_{Ti} = \rho_{Ti} \cdot V_{Ti} = 0.866$ kg;
$m_{Al} = \rho_{Al} \cdot V_{Al} = 0.814$ kg.

Actual safety margins (for comparison)

$n_{1.act} = 3.12$; $n_{2.act} = 3.32$;
$n_{1.RP} = 3.05$; $n_{2.act} = 3.16$;
$n_{1.act.Ti} = 3.04$; $n_{2.act.Ti} = 3.45$;
$n_{1.act.Al} = 3.18$; $n_{2.act.Al} = 3.14$.

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Conclusion.

Structure made of **steel round pipe** has the same material cost as a structure made of **steel bar**, but it’s **weight** is only **96% of the steel bar structure**, while it provides sufficient safety margin.

Structures made of **titanium round bar** and **aluminum round bar** have significantly lower weight – **17%** and **16% of the steel bar structure weight**, respectively. Titanium structure is more expensive – **141% of steel bar structure and aluminum is just 66% of steel bar structure**. At the same time, the safety margin is sufficient.

The final decision must be made from **budget or lightweight limitations**.

Thank you for your attention!

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